

THE THEORY OF COMMODITY PRICE STABILISATION RULES: WELFARE IMPACTS AND SUPPLY RESPONSES*

Commodity price stabilisation schemes are again topical, this time as a result of pressure from UNCTAD for an integrated programme on commodities as part of the 'New International Order'. Despite the long history of interest in the subject and its evident importance for developing countries the conventional analysis of the impact of price stabilisation is remarkably simplistic.¹ The theory was developed by Waugh (1944), Oi (1961), and Massell (1969) and provides the basis for most subsequent theoretical investigations (of which some are presented in the references and surveyed in Turnovsky, 1978*b*) and empirical estimates (e.g. MacAvoy and McNicol, 1976; World Bank, 1977). The Waugh-Oi-Massell approach assumes 'linear demand and supply schedules, instantaneous reaction of supply and demand to changes in market prices, additive stochastic disturbances and price stabilisation at the mean of the prices which would have prevailed in an unstabilised market' (World Bank, 1977), as shown in Fig. 1.

Additive stochastic disturbances are the natural specification for the estimation of linear demand and supply equations such as

$$\left. \begin{aligned} Q^d &= a - bp + \tilde{u} && \text{demand,} \\ Q^s &= c + dp + \tilde{v} && \text{supply,} \end{aligned} \right\} \quad (1)$$

where \tilde{u} , \tilde{v} are stochastic error terms. In such a case the inverse demand and supply schedules graphed in Fig. 1 vary without altering their slope, so that S_2S_2 is parallel to S_1S_1 but horizontally displaced.

Most of the 'core' commodities which UNCTAD is concerned to stabilise are agricultural commodities for which this model is most unsatisfactory, for the following reasons:

(i) Additive disturbances imply that bad weather has the same absolute impact on supply regardless of the acreage of the crop planted, whereas a more natural specification would make disturbances *proportional* to potential yield.²

(ii) Linearity implies that it is feasible to stabilise prices at their mean, in which case average supply will not change. If, as seems likely, demand or supply schedules are non-linear such price stabilisation will be infeasible – the buffer

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¹ With a few exceptions, such as Samuelson (1972) and Turnovsky (1976).

² A point made with convincing arguments by Hazell and Scandizzo (1975).

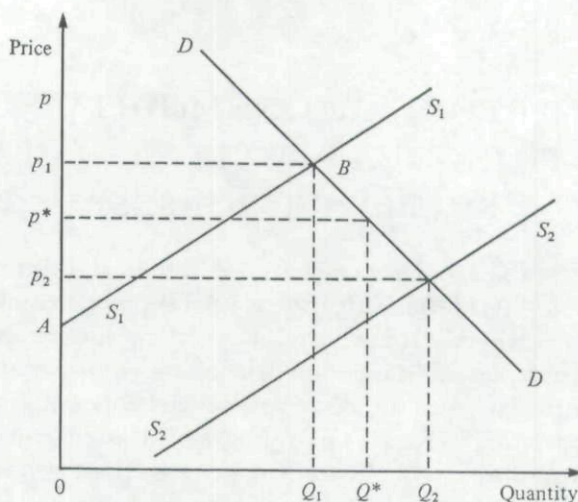


Fig. 1. Price stabilisation with additive risk.

stock would steadily accumulate or decumulate. We shall argue that the emphasis on prices has been misplaced, and that it is more logical to work in terms of quantities.¹

(iii) There is no distinction made between *ex-ante* and *ex-post* supply curves. In practice, a large part of the costs of producing crops are incurred before the weather is known, and certainly before the market price has been established. The supply schedule at harvest time is thus virtually vertical, with its location depending both on the weather, and on the level of inputs already chosen. It is therefore inappropriate to measure producers' profits as the area between the supply schedule and the price line (e.g. area Ap_1B in Fig. 1) as this requires costs to be the area below the supply schedule ($OABQ_1$) and for these to vary with the weather. It might be relatively simple to relate *ex-post* supply to planned or *ex-ante* average supply, \bar{Q} , by, for example, replacing equation (1) with

$$Q^s = \bar{Q} + \tilde{u}$$

but in general it is not possible to express average supply as a function of the expected price, since producers choose inputs with an eye to the expected *return*, which is in general not equal to the product of expected output and expected price. The model is therefore misleading both as a positive description of market equilibrium and for a normative measure of producer benefit.

(iv) The welfare analysis of consumers is similarly suspect when shifts in demand are a contributory cause of variability. A fall in demand leads to a decrease in the area of the Marshallian triangle which is taken as the measure of consumers' welfare, but if the fall is a result of a decrease in the price of a substitute then consumers will in fact be better off. Unless the source of price variability is carefully specified it is impossible to measure its impact on consumers.

¹ Waugh (1966) and Wegge, Sosnick and George (1971) consider the case where the price is stabilised at some weighted average, whilst Samuelson (1972) argues the general infeasibility of stabilising at the mean price.

(v) Most of the advocates of price stabilisation argue that reducing price variability is good in itself. If this leads to reduced income variability with no change in the mean income of producers then production risk will fall and risk averse producers will indeed be better off. However, *income* variability will not necessarily decrease with *price* stabilisation (especially if the elasticity of demand is low) and there is no guarantee that mean incomes will remain unchanged. Even if all these conditions are satisfied, the measurement of Marshallian triangles completely ignores this supposedly major benefit.

(vi) The conventional analysis contrasts no stabilisation with perfect, costless price stabilisation. Perfect price stability would require an infinite buffer stock to remain indefinitely feasible, and would thus be infinitely costly. In general, incomplete stabilisation will be desirable, but the model is not well suited to analysing partial stabilisation.¹

(vii) If, as is likely, price stabilisation changes the average return then producers will gradually learn of this and adjust their inputs. The long run impact of stabilisation may be quite different from its immediate impact as a result of this supply response. The model, however, predicts no change in average supply, \bar{Q} .

(viii) If farmers differ in their attitudes to risk then price stabilisation may affect them differentially, for it will affect both average incomes and their riskiness, possibly in different directions. The model is squarely in the Marshallian tradition of the representative producer, and is therefore silent on distributional issues.

It is the aim of this paper to suggest an alternative framework for the analysis of commodity stabilisation schemes, based on more secure micro-economic foundations. We begin with what might be termed the general theory of partial (or incomplete) price stabilisation. This meets objection (vi) above, and shows how the shape of the demand schedule and the source and specification of risk influence the size and distribution of welfare gains. It therefore allows the reader to appreciate the importance of the more detailed model specification which is required to investigate the remaining questions. This new model allows one to distinguish between the short run and long run impact of stabilisation, and to examine the importance of risk aversion and individual supply elasticity on the distribution of gains and losses from partial stabilisation.

Obviously, it is impossible to provide a complete theory of commodity price stabilisation within the confines of a single, brief paper, and the work presented here is indeed a small (but central) part of a more comprehensive study. In particular, we do not discuss the dynamics of price stabilisation in the paper (price expectations, learning, the stochastic nature of buffer stocks, etc.), nor do we model demand uncertainty, the macro-economic impact of risk and stabilisation, market imperfections, interactions with future markets, private speculators, with other commodities, and a host of other important issues. For these, and for a more detailed exposition of some of the key concepts presented here, the

¹ Turnovsky (1978a) examines a simple parameterisation of a partial stabilisation scheme, but restricts the analysis to the linear Waugh-Oi-Massell case.

interested reader is referred to our forthcoming book (Newbery and Stiglitz, 1980). Finally, we should point out that the buffer stock rules analysed here are not optimal rules, which can only be derived from a complete dynamic analysis, as derived and discussed in the book.

I. THE GENERAL THEORY OF PARTIAL PRICE STABILISATION

A price stabilisation scheme is any programme that leads to a reduction in the dispersion of price. Several measures or definitions of price dispersion have been employed, notably that of the coefficient of variation, but Rothschild and Stiglitz (1970) have shown these to have important limitations. They introduced the concept of *mean-preserving spreads* and demonstrated the coherence this gave to the study of risk. The key issue is which variable to use in the analysis. Most writers have implicitly used a mean-price preserving decrease in price variability, but argument (ii) above showed this to be inappropriate because it is infeasible. The appropriate concept is a *mean-quantity preserving decrease in price dispersion*, which is feasible and easy to interpret. It is affected by transferring (by storage, which we assume occurs without wastage) a unit of output from a date at which price is low to a date when it is high. Thus, if $p_i = p_i(Q^c)$ is the price at date i when consumption is Q^c , and if

$$p_1 > p_2$$

the price distribution generated by

$$Q_i^c = \begin{cases} Q_i^c & i \neq 1, 2 \\ Q_1^c + \delta Q & i = 1 \\ Q_2^c - \delta Q & i = 2 \end{cases} \quad (2)$$

is less dispersed than the original distribution, provided δQ is small enough and if demand at i is independent of price at j . (Demand for storage, as opposed to consumption, will obviously depend on prices at different dates. Since we are assuming that only the buffer agency engages in storage it seems reasonable to assume intertemporally independent demand, though for a full analysis of commodity stabilisation schemes the responses of private speculators should be investigated. See Newbery and Stiglitz, 1980.)

Fig. 2 illustrates the effect of a mean output preserving transfer on the price distribution. The probability density functions are graphed on the axes, and show the effect of a shift of δQ from Q_2 to Q_1 . Q_2 occurs less often, because it becomes $Q_2 - \delta Q$, and Q_1 also occurs less often, becoming $Q_1 + \delta Q$, hence compressing the distribution whilst leaving the mean unchanged. The price distribution is derived from the supply distribution via the demand curve.¹

¹ We assume the demand schedule is stable and downward sloping. If demand is variable a less disperse price distribution may correspond to a more disperse distribution of consumption.

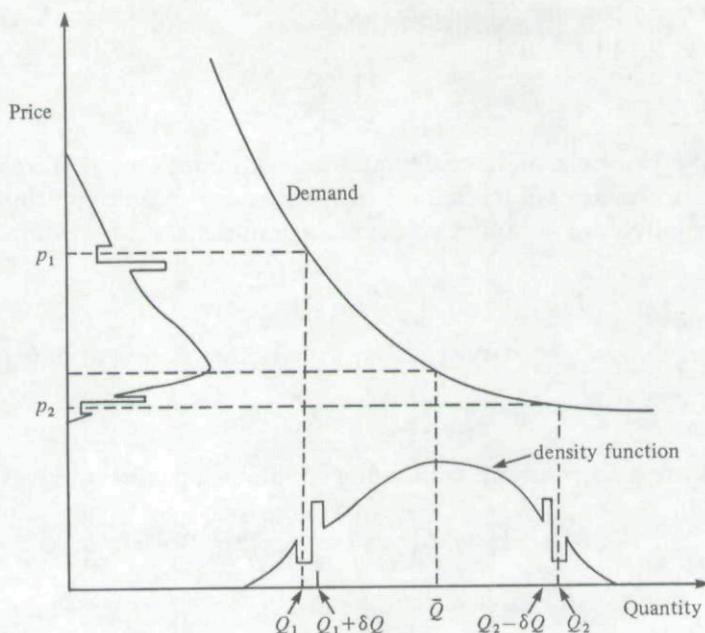


Fig. 2. Mean output preserving decrease in variability.

II. THE WELFARE EFFECT OF STABILISATION

We ask what effect this transfer has on the three interested parties: consumers, the storage agency, and producers. Of these, the easiest to measure is the effect on the income of the buffer stock, which is at date t

$$Y_t^b = (Q_t^c - Q_t^s) p(Q_t^c).$$

Q^s is supply, which varies from date to date, but is assumed unaffected by the transfer, whilst Q^c is consumption, which will be affected by the actions of the buffer agency, as will the market clearing price. The impact on the buffer stock of the transfer from date 2 to date 1 is

$$\delta Y^b = \left. \frac{dY^b}{dQ^c} \right|_1 \delta Q - \left. \frac{dY^b}{dQ^c} \right|_2 \delta Q$$

or, more briefly

$$\delta Y^b = \Delta \left[\frac{dY^b}{dQ^c} \right] \delta Q,$$

where Δ is the difference operator which takes the difference of the value in brackets between dates 1 and 2. Hence

$$\delta Y^b = \Delta[p] \delta Q + \Delta \left[(Q^c - Q^s) \frac{dp}{dQ} \right] \delta Q, \quad (3)$$

assuming zero storage and interest costs. (Such costs will, of course, have to be estimated if the optimal amount of price stabilisation is to be identified. See Newbery and Stiglitz (1980) for a full treatment of optimal stockholding rules.)

For producers the transfer of δQ makes no difference to their sales, Q^s , but it does alter the price by an amount

$$\frac{dp}{dQ} \delta Q = -\frac{p}{\epsilon Q^c} \delta Q, \quad (4)$$

where ϵ is the price elasticity of demand for consumption, Q^c (measured as a positive number). Since we are holding mean quantity constant producers do not change their purchases of inputs, so the change in their utility is simply

$$\delta u = \Delta \left[u'(y) \frac{\partial y}{\partial p} \frac{dp}{dQ} \right] \delta Q, \quad (5)$$

where $u'(y)$ is the marginal utility of farmer's income, y , and at date i

$$\left(\frac{\partial y}{\partial p} \right)_i = Q_i^s.$$

Since inputs are held constant, combining equations (4) and (5) gives

$$\delta u = - \left[\Delta u' E \left(\frac{p Q^s}{\epsilon Q^c} \right) + \Delta \left(\frac{p Q^s}{\epsilon Q^c} \right) E u' \right] \delta Q$$

or the cash value of producer's benefit¹ is

$$B = \frac{\delta u}{E u'} = - \left[\frac{\Delta(u')}{E u'} E \left(\frac{p Q^s}{\epsilon Q^c} \right) + \Delta \left(\frac{p Q^s}{\epsilon Q^c} \right) \right] \delta Q, \quad (6)$$

where E is the expectations operator, taking the average of the values of terms to its right at the two dates.² The first term is the *risk benefit*, the benefit of transferring income from periods of low to periods of high marginal utility, whilst the second term is the *transfer effect*, the value of the change in average income received.

The remaining participants are consumers, whose welfare is best measured by the indirect utility function $V(p, I)$, where I is consumer income. As before

$$\delta V = \Delta \left(\frac{dV}{dQ^c} \right) \delta Q = \Delta \left(\frac{\partial V}{\partial p} \frac{dp}{dQ^c} \right) \delta Q.$$

By Roy's identity

$$\frac{\partial V}{\partial p} = -\frac{\partial V}{\partial I} Q^c = -Q^c V_I$$

with the obvious subscript notation for partial derivatives, so consumer benefits, B^c , are

$$B^c = \frac{\delta V}{E V_I} = \Delta \left(-Q^c \frac{dp}{dQ^c} \right) \delta Q + \frac{\Delta(V_I)}{E V_I} E \left(\frac{p}{\epsilon} \right) \delta Q. \quad (7)$$

All three expressions give the cash value of the welfare effect of the price stabilising transfer, since they are deflated by the average value of the marginal utility of income, and to that extent they are comparable. Notice, first of all, the remarkable

¹ Defined as the amount the producer would be willing to pay to have the price variability reduced in the given way.

² Note that

$$\begin{aligned} \Delta(xy) &= x_1 y_1 - x_2 y_2 \\ &= \frac{y_1 + y_2}{2} (x_1 - x_2) + \frac{x_1 + x_2}{2} (y_1 - y_2) \\ &= \Delta(x) E y + \Delta(y) E x. \end{aligned}$$

result, that if we ignore the terms involving changes in marginal utility ($\Delta u'$ and ΔV_I), then the sum of all three benefits is simply

$$\delta Y^s = \Delta(p) \delta Q.$$

Thus, if we would ignore the effect of stabilisation on the marginal utility of incomes (that is, the distributional impacts) the benefits of price stabilisation are simply the benefits of arbitrage, that is, of moving consumption from dates of low value (low price) to dates of high value. If this could be achieved costlessly, then perfect price stabilisation is desirable, but since it is both costly and ultimately infeasible, perfect stabilisation is neither desirable nor possible.

There is another way of expressing the benefits of stabilisation. The sum of changes in producer and buffer stock income is

$$\delta Y^b + \delta y = \Delta \left(p + Q^c \frac{dp}{dQ^c} \right) \delta Q = \delta X^c,$$

where $X^c = pQ^c$ is consumers' expenditure on the commodity. If we notionally aggregate buffer profits with producers then any increase in producers' income is matched by an equal increase in consumers' expenditure. If we again ignore the effect of change in price on the marginal utility of consumers' income¹ (the last term of equation (7)), then

$$B^c = -\delta X^c + \Delta(p) \delta Q.$$

Thus the cash value of price stabilisation to consumers (dividing by the marginal utility of income to express the result in money terms) is the sum of the transfer to producers and the buffer stock plus an *arbitrage effect* of transferring goods from dates of low value to dates of high value.

Notice that we cannot sign the overall impact on consumers without specifying the functional form. This is in contrast to the general view which states that since the indirect utility function is convex in prices, price stabilisation harms consumers. The result depends on an unchanged mean price, whereas our basic contention is that there is no reason to expect mean price to remain constant.

These various general formulae provide the basis for the detailed analysis of special cases. Consider first the effect of a little stabilisation, starting from no stabilisation.

(1) *The effects of a small amount of stabilisation*

Initially supply equals demand, so $Q^s = Q^c$. The transfer term from producers is then

$$-\Delta \left(\frac{p}{\epsilon} \right) \delta Q, \quad (8)$$

which is negative if the elasticity of demand is constant. If, on the other hand, demand is linear and there is no change in the slope of the demand curve, $-b$ (additive risk), the transfer term is

$$\Delta \left(Q \frac{dp}{dQ} \right) \delta Q = -b \Delta(Q) \delta Q > 0.$$

¹ I.e. we assume $V_{I,p}$ is negligible. This is reasonable if X^c/I is small, and for most primary commodities this ratio is less than 1%.

This immediately demonstrates the importance of choosing between linear and log-linear specifications. Either way, the cash value of the transfer effect is exactly equal to and opposite in sign to the cash value to consumers. There remain as net benefits the buffer profits and the risk benefit. The latter is found from the expression for the marginal utility of income, itself a function of supply (= demand). Expanding about mean quantity, \bar{Q} , gives

$$u'[y(Q)] \cong u'[y(\bar{Q})] + (Q - \bar{Q})u''[y(\bar{Q})]\frac{dy}{dQ}.$$

If the degree of relative risk aversion is R ,¹ where

$$R \equiv -\frac{yu''(y)}{u'(y)}, \quad y = p(Q)Q$$

then

$$u'(Q) = u'(\bar{Q}) \left[1 - R \left(1 - \frac{1}{\epsilon} \right) \frac{Q - \bar{Q}}{\bar{Q}} \right],$$

so, if R and ϵ are constants,

$$-\Delta(u') = u'[y(\bar{Q})] R \left(1 - \frac{1}{\epsilon} \right) \frac{\Delta Q}{\bar{Q}},$$

and the risk benefit is, for the constant elasticity case

$$R \left(1 - \frac{1}{\epsilon} \right) \frac{\bar{p}}{\epsilon \bar{Q}} \Delta Q \delta Q, \quad \bar{p} \equiv Ep,$$

whose sign depends on whether demand is elastic or not. The transfer effect, in the constant elasticity case, is

$$-\frac{1}{\epsilon} \Delta(p) = \frac{p}{\epsilon^2 Q} \Delta(Q^c) \quad (9)$$

which is always negative. The total impact on producers is

$$B = \frac{\bar{p}}{\epsilon \bar{Q}} \left[R \left(1 - \frac{1}{\epsilon} \right) + \frac{1}{\epsilon} \right] \Delta(Q^s) \delta Q. \quad (10)$$

Thus unless risk aversion is large and demand inelastic, the net effect will be negative.

(2) *The effects of a small destabilisation from perfect stabilisation*

Here the simplification changes from $Q^s = Q^c$ to $p = \hat{p} = p(\bar{Q})$, $Q^c = \bar{Q}$, provided that the source of variability is on the supply side. The producers' transfer term is now

$$-\frac{\hat{p}}{\bar{Q}} \Delta \left(\frac{Q^s}{\epsilon} \right) \delta Q. \quad (11)$$

¹ Strictly speaking we should distinguish between *revenue*, pQ , and *income*, which includes sources other than the sale of the crop, and is reduced by the purchase of inputs. The model can thus be interpreted as one where only one crop is grown with no purchased inputs, only own labour. It is possible to extend the analysis to include other sources of income and purchased inputs, but at considerable cost of lost simplicity.

If the demand elasticity is constant, a slight amount of destabilisation lowers revenue (notice the sign change as we are now making transfers in the opposite direction to *reduce* stability). A graph of average producer revenue against stabilisation will be U-shaped, as in Fig. 3 below. In terms of average revenue any amount of stabilisation will be unattractive unless complete stabilisation is preferable to no stabilisation; that is, unless (with no demand risk)

$$EQp(EQ) > EQp.$$

For this to hold, revenue pQ must be a concave function of sales, Q , which, if ϵ is constant, requires $\epsilon > 1$. If, however, demand is inelastic, no amount of stabilisation will raise producers' average revenue.

There remains the risk benefit, which is now

$$-\frac{\hat{p}}{\epsilon} \Delta \frac{[u'(\hat{p}Q)] \delta Q}{Eu'}.$$

In this case the Taylor series expansion of u' gives

$$u'(\hat{p}Q) \cong u'(\hat{p}\bar{Q}) \left(1 - R \frac{Q - \bar{Q}}{\bar{Q}} \right)$$

so the risk benefit is worth

$$\frac{R\hat{p}}{\epsilon\bar{Q}} \Delta(Q) \delta Q \quad (12)$$

and slight destabilisation increases the risk benefit, offsetting the transfer effect. The net effect on producers is

$$B = \frac{\hat{p}}{\epsilon} (R - 1) \frac{\Delta(Q)}{\bar{Q}} \delta Q, \quad (13)$$

whose sign depends on the magnitude of risk aversion, R . As expected, if producers are very risk averse, slight destabilisation makes them worse off.

(3) *The allocation of buffer profits*

The conventional approach only distinguishes two parties affected by stabilisation, consumers and producers. This is reasonable when contrasting perfect (costless) stabilisation with no stabilisation since at each extreme the buffer stock makes no profits. With partial stabilisation this is no longer true, and it is sometimes convenient to preserve the simple dichotomisation between producers and consumers to allocate buffer profits to one or other side. It seems more natural to allocate them to producers, as the transfer term which appears in the consumers' benefit is a transfer to producers and the buffer stock taken together. It does, however, make a potentially substantial difference to the evaluation of the producer benefits to include buffer profits, as can be seen by estimating them for a small amount of stabilisation. Adding together equations (3) and (8) gives a change in producer revenue of

$$\delta Y = \left(1 - \frac{1}{\epsilon} \right) \Delta(\hat{p}) \delta Q$$

when ϵ is constant, which is positive if demand is elastic. Recall from equation (9) that if buffer profits are not included, and the elasticity is constant, then the

transfer effect is always negative. A small amount of destabilisation does not generate any buffer profits, so the effect of stabilisation on producers' revenue is as graphed below. The dotted lines include buffer profits, the continuous lines exclude them.

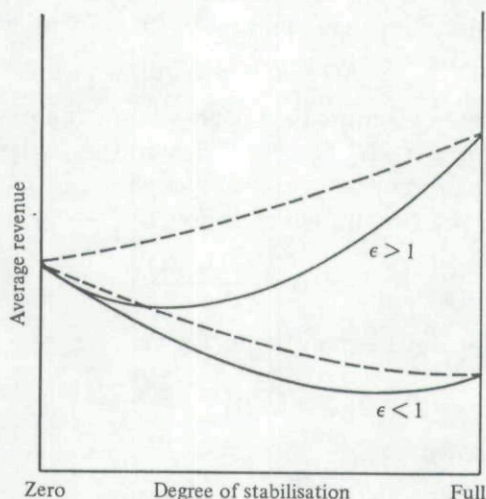


Fig. 3. The effect of stabilisation on product revenue.

III. THE SUPPLY RESPONSE TO STABILISATION

The previous section developed a method for analysing the impact of reduced price dispersion on consumers, producers and the buffer stock on the assumption of no supply response. As such, it is useful for studying the immediate impact of any proposal, but it is obviously also important to examine the long-run effects, allowing supply to respond. The central question is whether the long-run effects are in the same direction as, or the opposite direction to, the short-run impact. If they are in the same direction then the very general approach set out above provides the appropriate qualitative guide, whilst if the long-run effect reverses the short-run impact the analysis of price stabilisation becomes somewhat delicate.

To model supply responses, it is necessary to become more explicit about the choices facing the farmer and his decision criterion. In so doing, it becomes possible to explore other questions, particularly the distributional consequences of price stabilisation. It also becomes necessary to specify what is meant by the degree of price stabilisation. The model is initially developed to study the effect of price stabilisation when the source of price variability lies on the supply side, but is extended to show how to analyse other schemes (such as income stabilisation) in the presence of demand as well as supply variability.

(1) *A model of supply under price risk*

We begin the analysis with the following simple model. Farmers grow only a single crop. All farmers face identical, multiplicative production risk and have

access to the same production possibilities, so that output per farmer, q , is a linear¹ stochastic function of input of effort, x :

$$q = \theta x, \quad E\theta = 1,$$

where θ is the random effect of weather, and the source of the randomness in prices. Total output Q is thus

$$Q = \theta \bar{Q}, \quad \bar{Q} = \Sigma x, \quad (14)$$

where \bar{Q} is average output, the sum of individual farmer's average output, x . Price is a stable function of the random total output, $p(Q)$, and is hence random. The welfare of the farmer depends both on his income, y , and the effort he has expended, x :

$$\left. \begin{aligned} U &= U(y, x), \\ y &= p(Q) \theta x. \end{aligned} \right\} \quad (15)$$

The farmer chooses how much effort to supply before the weather is known and aims to maximise expected utility, EU , which, for simplicity, is assumed to be additively separable:

$$EU(y, x) = Eu(y) - v(x), \quad (16)$$

where $u'' < 0$, i.e. the farmer is risk averse.

Equation (15) implies that, for the individual farmer, income risk is also multiplicative with the random factor $p\theta$, but obviously for society this is not true. The individual farmer maximises expected utility taking the price distribution as given, but he is aware that p depends on θ , and that good weather with high personal production will be associated with low prices. In short, farmers are assumed to hold *rational expectations*, which Muth (1961) defines as those which would be predicted by the relevant economic theory, given the information currently available. The reason for this assumption is that we are interested in the long-run supply response to stabilisation, once farmers have learnt how the price distribution has changed. In the short run it is probably more appropriate to assume that supply does not respond as in the previous section. Turnovsky (1974) has discussed the case in which farmers form naïve forecasts on the basis of past observed prices, but the main difficulty in analysing such inefficient uses of information lies in predicting how the inefficiencies will respond to a *change* in the market environment. The reader is referred to Newbery and Sitglitz (1980) for a more extensive discussion of these issues.

The farmer chooses input x , yielding average welfare W :

$$W = \underset{x}{\text{Max}} \{Eu[p(\theta \bar{Q}) \theta x] - v(x)\}, \quad (17)$$

so that

$$Eu'(y) p\theta = v'(x). \quad (18)$$

This can be solved to find the farmer's supply of effort as a function of the price distribution. A natural question to ask is how average output, Eq , depends on the *level* of prices. (This is the nearest equivalent to a supply curve, for, with price

¹ Diminishing returns to effort and increasing disutility of effort are indistinguishable in this model, so we choose the simplest specification.

variability, there is no longer any simple relationship between price and quantity supplied.) For an individual farmer, if the whole price distribution shifts upwards, then presumably he will wish to supply more. We can trace out a pseudo-supply curve for the individual farmer by setting

$$p = \lambda D(Q) = \lambda D(\bar{Q}\theta)$$

and allowing λ to vary, holding \bar{Q} constant.¹ If there is no production risk, this is exactly the same as asking how supply will depend on price, or solving for q as a function of price. The natural measure of the elasticity of supply, η , is

$$\eta \equiv \frac{\lambda}{Eq} \frac{dEq}{d\lambda} = \frac{\lambda}{x} \frac{dx}{d\lambda}. \quad (19)$$

Differentiate the logarithm of equation (18) with respect to λ :

$$\frac{E\left(u' \frac{p\theta}{\lambda} + p\theta u'' \frac{dy}{d\lambda}\right)}{Eu' p\theta} = \frac{xv''}{v'} \frac{1}{x} \frac{dx}{d\lambda}, \quad (20)$$

where

$$\frac{1}{y} \frac{dy}{d\lambda} = \frac{1}{\lambda} + \frac{1}{x} \frac{dx}{d\lambda}.$$

Define the two elasticities R , γ :

$$R = -\frac{yu''}{u'}, \quad \gamma = \frac{xv''}{v'},$$

where R is as before the coefficient of relative risk aversion.² Equation (20) can be rewritten as

$$\frac{1}{\lambda} - \frac{1}{\lambda} \left(1 + \frac{\lambda}{x} \frac{dx}{d\lambda}\right) \left(\frac{ERu' p\theta}{Eu' p\theta}\right) = \frac{\gamma}{x} \frac{dx}{d\lambda},$$

or

$$\eta = \frac{1 - \bar{R}}{\gamma + \bar{R}}, \quad (21)$$

where

$$\bar{R} \equiv \frac{ERu' p\theta}{Eu' p\theta}.$$

(2) Homothetic stabilisation schemes

A stabilisation scheme changes the probability distribution of prices corresponding to a given distribution of outputs, so that the resulting price distribution becomes

$$p = p(\theta, \bar{Q}, z); \quad (22)$$

where z is some parameterisation of a family of stabilisation schemes, \bar{Q} is average output, and θ describes the underlying source of risk (which here is a scalar). For

¹ Note that $D(Q)$ is the non-stochastic demand curve, and that prices vary because Q varies. Hence the difficulty of relating average supply to 'the' price. The average price will be proportional to λ .

² But see footnote 1, p. 806 above.

example, the simplest linear buffer stock rule would be to purchase all production but to sell a more stable, weighted average of \bar{Q} , Q , to consumers:¹

$$Q^c = z\bar{Q} + (1-z)Q = \bar{Q}\phi(z) \quad (0 \leq z \leq 1), \quad (23)$$

$$\phi(z) \equiv z + (1-z)\theta.$$

The consumer price would then be

$$p = p[\bar{Q}\phi(z)] = p(\bar{Q}, \theta, z).$$

A *homothetic stabilisation scheme* is one which allows the price to be expressed as a product of a riskless and a risky term:

$$p = g(\bar{Q}, z) h(\theta, z). \quad (24)$$

This implies that the percentage change in price for a given change in average supply is independent of the value of the random variable θ . The linear stocking rule above gives a homothetic stabilisation scheme if the elasticity of demand is constant, with $g = \bar{Q}^{-1/\epsilon}$, $h = \phi^{-1/\epsilon}$. More generally, provided risk is multiplicative and demand of constant elasticity, it is always possible to design a homothetic stabilisation scheme no matter what the source of the risk, or the objective of the buffer (to stabilise prices or producer incomes).

Once the particular stabilisation scheme has been specified, equation (18) can be solved to give the equilibrium supply of effort (and hence average output) as a function of the level of stabilisation, z :

$$x = x(z).$$

Substituting this back into equation (17) gives the equilibrium average welfare as a function of z , $W(z)$. Differentiate this totally with respect to z , and use the envelope theorem (that $\partial W/\partial x = 0$) to obtain

$$\frac{dW}{dz} = Eu'\theta \frac{dp}{dz} x. \quad (25)$$

But, from equation (24), for a homothetic stabilisation scheme with an elasticity of demand ϵ ,

$$\frac{dp}{dz} = \frac{\partial p}{\partial z} + \frac{\partial p}{\partial \bar{Q}} \frac{d\bar{Q}}{dz} = \frac{\partial p}{\partial z} - \frac{p}{\epsilon} \frac{1}{\bar{Q}} \frac{d\bar{Q}}{dz}. \quad (26)$$

Stabilisation has therefore two effects – a direct effect in changing prices in each state of nature, measured by $\partial p/\partial z$, and an indirect effect, where stabilisation affects effort, which affects average output \bar{Q} , and hence price. Our basic question is whether this indirect effect can reverse the direct (or short run impact) effect. To answer this, differentiate the logarithm of equation (18) again, this time with respect to z :

$$\frac{1}{x} \frac{dx}{dz} = \frac{1-R}{R+\gamma} \frac{xEu'\theta (dp/dz)}{xEu'\theta p} = \frac{\eta}{xv'(x)} \frac{dW}{dz}, \quad (27)$$

¹ This may not be feasible if $Q < \bar{Q}$ and stocks are too low. The possibility that stocks may run out introduces an essential non-linearity which cannot be analysed in our framework. Instead the problem must be reformulated in a dynamic context, see Newbery and Stiglitz (1980). Meanwhile we assume adequate stocks for feasibility.

substituting from equations (21), (25) and (18), and on the assumption that risk aversion, R , is constant. Equation (27) gives a simple relationship between the supply response, the elasticity of supply, η , and the welfare impact. For the normal case of positive supply elasticity, when welfare is increased, supply increases, confirming intuition.

Equations (26) and (27) can be combined to give

$$\frac{1}{x} \frac{dx}{dz} + \frac{\eta}{\epsilon} \frac{1}{\bar{Q}} \frac{d\bar{Q}}{dz} = \eta \frac{Eu' \theta (\partial p / \partial z)}{Eu' \theta p} = \frac{\eta}{xv'(x)} \frac{\partial W}{\partial z}, \quad (28)$$

provided that the elasticity of demand, ϵ , is constant. If the supply characteristics of all farmers are similar, this reduces to

$$\frac{1}{\bar{Q}} \frac{d\bar{Q}}{dz} = \frac{1}{x} \frac{dx}{dz} = \frac{\epsilon \eta}{\epsilon + \eta} \frac{1}{xv'(x)} \frac{\partial W}{\partial z}, \quad (28')$$

which, when combined with equation (27), yields

$$\frac{dW}{dz} = \frac{\epsilon}{\epsilon + \eta} \frac{\partial W}{\partial z}. \quad (29)$$

Provided the 'supply curve' (average supply as a function of scale shifts in the demand curve) cuts the demand curve from below, the immediate and long run impacts of price stabilisation are in the same direction. This proviso is, however, just the condition for Walrasian stability, in a market with no uncertainty.¹

(3) *The effect on consumers*

The relationship between the impact and long run effects on consumers is particularly simple in the case of stable consumer demand, for, from equation (26),

$$\frac{dEV(p)}{dz} = \frac{\partial EV(p)}{\partial z} + \frac{\partial EV(p)}{\partial \bar{Q}} \frac{d\bar{Q}}{dz}. \quad (30)$$

The first term is the impact effect, the second is a transfer term corresponding to the change in the average supply. If producers and consumers are weighted equally, this will cancel out. A mean *quantity* preserving change in price dispersion is thus all that is needed to value the impact on total welfare in this special case (though it is inadequate to investigate the distributional impact). Roy's identity can again be used to evaluate equation (30):

$$\frac{1}{V_I} \frac{dEV}{dz} = - \left[EQ^c \frac{\partial p}{\partial z} - \frac{1}{\epsilon} E(Q^c p) \frac{1}{\bar{Q}} \frac{d\bar{Q}}{dz} \right] \quad (31)$$

provided that $V_{Ip} = 0$, or the marginal utility of income is roughly constant.

¹ See Samuelson (1947, p. 260). The appropriate stability condition for markets with risk depends on the formation of expectations about the whole of the probability distribution. (See Newbery and Stiglitz, 1980.) It is, for instance, a more stringent stability condition than cobweb stability conditions, for, if not satisfied there would be monotonic departures from equilibrium.

Thus the long run and impact effects on consumers may actually differ (e.g. if producers respond by reducing supply because of a fall in income).

(4) *The evaluation of particular schemes*

It is straightforward to evaluate a variety of stabilisation schemes, depending on the source of the risk and the objective of the scheme. As an example, consider the price stabilisation scheme of equation (23).

$$\frac{\partial p}{\partial z} = \frac{p(\theta - 1)}{\epsilon \phi}, \quad \phi \equiv z + (1 - z)\theta,$$

so

$$\frac{\partial W}{\partial z} = \frac{x}{\epsilon} Eu' \theta p(\theta - 1) / \phi, \quad x = Eq = \bar{q}. \quad (32)$$

(5) *A small degree of stabilisation*

At $z = 0$, $\phi = \theta$, and a Taylor series expansion of the right-hand side of equation (32) gives, for the case of constant elasticity ϵ , and constant risk aversion, R , the short-run impact as:

$$\left. \begin{aligned} \frac{1}{u'} \frac{\partial W}{\partial z} &= \frac{p(\bar{Q}) \bar{q}}{\epsilon} \left[(1 - R) \left(1 - \frac{1}{\epsilon} \right) - 1 \right] \sigma^2, \\ B &= -\frac{p(\bar{Q}) \bar{q}}{\epsilon} \left[R \left(1 - \frac{1}{\epsilon} \right) + \frac{1}{\epsilon} \right] \sigma^2, \end{aligned} \right\} \quad (33)$$

where $\sigma^2 = E(\theta - 1)^2$, the variance of θ . (That is, σ is the coefficient of variation of output.) Equation (33) is the same as equation (10), confirming the generality of the result.

If all producers are alike, the second term of equation (31) can be derived from equation (28'):

$$\left. \begin{aligned} \frac{1}{\bar{Q}} \frac{d\bar{Q}}{dz} &= \frac{\eta}{\epsilon + \eta} \frac{Eu' p(\theta - 1)}{Eu' p\theta}, \\ \frac{1}{\bar{Q}} \frac{d\bar{Q}}{dz} &= -\frac{\eta}{\epsilon + \eta} \left[R \left(1 - \frac{1}{\epsilon} \right) + \frac{1}{\epsilon} \right] \sigma^2. \end{aligned} \right\} \quad (34)$$

The supply response to a small amount of stabilisation is thus likely to be negative, which explains why the long-run effect on producers is not so disadvantageous as the immediate impact.

The immediate impact on consumers is the first term in equation (30) which is positive

$$B^c = -\frac{1}{\epsilon} Ep Q^c(\theta - 1) / \theta = \bar{p} Q \frac{\sigma^2}{\epsilon^2} = \bar{X} \sigma_p^2, \quad (35)$$

where X is expenditure and σ_p is the coefficient of variation of prices. The long-run impact is given by equations (30) and (35) and is

$$B^c = \frac{\bar{X}}{\epsilon + \eta} [\epsilon - \eta R(1 - \epsilon)] \sigma_p^2,$$

which is positive or negative as $R(1 - \epsilon) \leq \epsilon/2$.

The change in average buffer profits Y^b is found as follows. If

$$Y^b = Ep(Q^c - Q) = z\bar{Q}Ep(1 - \theta),$$

$$\frac{1}{\bar{Q}} \frac{dY^b}{dz} = Ep(1 - \theta) + z \left[E(1 - \theta) \frac{\partial p}{\partial z} + E(1 - \theta) \frac{\partial p}{\partial \bar{Q}} \frac{d\bar{Q}}{dz} \right]. \quad (36)$$

At $z = 0$ this reduces to

$$\frac{dY^b}{dz} = \bar{Q}p(\bar{Q}) \frac{\sigma^2}{\epsilon} = \frac{\partial Y^b}{\partial z}.$$

If these profits are attributed to producers equation (33) is replaced by

$$\frac{1}{u'} \frac{\partial W}{\partial z} = \frac{p(\bar{Q}) \bar{q}}{\epsilon^2} (1 - R) (\epsilon - 1) \sigma^2,$$

which could easily be of the opposite sign to that excluding profits.

(6) *Small departures from complete stabilisation*

At $z = 1$, $\phi = 1$, $p = p(\bar{Q})$ and the short-run impacts are approximately

$$\frac{1}{u'} \frac{\partial W}{\partial z} = \frac{p(\bar{Q}) \bar{q}}{\epsilon} (1 - R) \sigma^2 > 0 \quad \text{if } \eta > 0,$$

$$\frac{\partial EV}{\partial z} = 0.$$

The supply response is

$$\frac{1}{\bar{Q}} \frac{d\bar{Q}}{dz} = \frac{\eta}{\epsilon + \eta} (1 - R) \sigma^2 > 0,$$

so the long-run impact on consumers is

$$\frac{(1 - R) \eta \epsilon}{\eta + \epsilon} \bar{X} \sigma_p^2 \geq 0 \quad \text{as } R \geq 1.$$

At complete stabilisation the welfare impact and supply response are thus typically in the opposite direction to those produced by a small amount of stabilisation. The effect on buffer income is found from equation (36) setting $p = p(\bar{Q})$, constant, and $z = 1$:

$$\frac{dY^b}{dz} = -\bar{Q}p(\bar{Q}) \frac{\sigma^2}{\epsilon} = \frac{\partial Y^b}{\partial z} < 0,$$

which, if attributed to producers, gives a short-run impact of

$$\frac{1}{u'} \frac{\partial W}{\partial z} = -\frac{p(\bar{Q}) \bar{q}}{\epsilon} R \sigma^2 < 0,$$

exactly reversing the direction of the short-run impact.

IV. THE DISTRIBUTIONAL IMPACTS OF STABILISATION WITH DIVERSE PRODUCERS

Land holdings are typically very unequally distributed amongst farmers, and it may therefore be very misleading to characterise the supply response to stabilisation schemes in terms of a representative farmer. It is important to

know whether farmers with different characteristics (like risk aversion) are affected differently by stabilisation. Equations (25) and (26) give

$$\frac{dW}{dz} = \frac{\partial W}{\partial z} - \frac{1}{\epsilon} \frac{\bar{q}}{\bar{Q}} \frac{d\bar{Q}}{dz} Eu' \theta p.$$

Suppose most farmers are alike, and dominate in determining the supply response, but a few are different. Let asterisks denote the dominant producers, and consider a small degree of stabilisation.

$$\begin{aligned} \frac{1}{\bar{y}^* u'^*} \frac{dW^*}{dz} &= - \left(\frac{1}{\epsilon + \eta} \right) \left[R^* \left(1 - \frac{1}{\epsilon} \right) + \frac{1}{\epsilon} \right] \sigma^2, \\ \frac{1}{\bar{y} u'} \frac{dW}{dz} &= - \left(\frac{1}{\epsilon + \eta} \right) \left\{ \left(1 - \frac{1}{\epsilon} \right) \left[R + \frac{\eta}{\epsilon} (R - R^*) \right] + \frac{1}{\epsilon} \right\} \sigma^2. \end{aligned}$$

It is evidently possible for more-risk averse farmers to be affected in the opposite way to less-risk averse farmers, and even for the long-run impact on the non-risk averse farmers to be in the opposite direction from the immediate impact.

V. DEMAND INDUCED VARIABILITY

The analysis follows closely along the lines of the previous section except now we have to specify the nature of the stochastic variations in the demand functions and the form of the stabilisation scheme. As an example, suppose that in addition to the supply risk of the previous section the demand function is

$$Q^c = \psi p^{-\epsilon}, \quad E\psi = 1,$$

where ψ is a random factor (perhaps generated by variations in other prices, or in income). A stabilisation scheme which sells

$$Q^c = \bar{Q} \theta^{1-z} \psi^z,$$

so that

$$p = (\psi/\theta)^{(1-z)/\epsilon} \bar{Q}^{-(1/\epsilon)}$$

is homothetic and as z varies from 0 to 1 gradually introduces complete price stabilisation. The previous general formulae stand, except that

$$\frac{\partial p}{\partial z} = \frac{p}{\epsilon} \log (\theta/\psi)$$

so that, for example,

$$\frac{\partial W}{\partial z} = \frac{1}{\epsilon} Eu' \theta p \log (\theta/\psi).$$

Obviously the correlation between θ and ψ will affect the outcome. If $\theta \equiv 1$ so that there is only variability in demand,

$$\frac{1}{\bar{y} u'} \frac{\partial W}{\partial z} = - (1 - R) (1 - z) \frac{\sigma^2}{\epsilon^2},$$

where $\sigma^2 = \text{Var } \psi$. This can also be expressed in terms of the *remaining* price variability, σ_p (the coefficient of variation)

$$\frac{1}{\bar{y}\bar{u}'} \frac{\partial W}{\partial z} = -\frac{1-R}{1-z} \sigma_p^2.$$

Income stabilisation schemes can likewise be analysed. Thus, if demand and supply variability are as before, the buffer sells

$$Q^c = \bar{Q} \theta^{1+\varepsilon(\varepsilon-1)} \psi^{\varepsilon}$$

and farmer's income is then

$$y = p \theta x = x p(\bar{Q}) [\theta^{(1-1/\varepsilon)} \psi^{(1/\varepsilon)}]^{1-\varepsilon}.$$

VI. CONCLUSION

We have developed a very general theory of partial price stabilisation which applies if mean supply does not change, and which illuminates the importance of common simplifying assumptions about the shape of the demand schedule and the source and nature of price variability. We then introduced the notion of a homothetic price stabilisation scheme to show how the supply response would modify this initial impact in the context of a more specific (but still fairly general model). If risk is multiplicative and demand and utility functions have constant elasticity the full impact of price stabilisation on identical producers is a simple positive fraction of the immediate impact (before supply adjusts), the fraction being $\varepsilon/(\varepsilon + \eta)$, where ε is the elasticity of demand, and η is the underlying supply elasticity. If producers are not identical, then it is still possible to analyse the long-run impact, but variations in attitudes to risk will have distributional consequences.

The assumption of constant elasticity utility functions played a critical role in the analysis. Elsewhere, we show that in the more general case, the short-run and long-run impacts may differ not only quantitatively (as here) but also qualitatively.

Our theoretical analysis has suggested that it may be difficult to assess the desirability of buffer stock schemes without knowledge of certain critical parameters, such as the elasticity of effort response and the elasticity of demand. We showed how, with knowledge of these parameters, the benefits (possibly negative) of price stabilisation, to both producers and consumers may be quantified.

Churchill College, Cambridge
All Souls College, Oxford

D. M. G. NEWBERY
J. E. STIGLITZ

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